The Oil Drum: Europe
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The limit of the statistic R/P in models of oil discovery and production

Posted by Chris Vernon on January 22, 2008 - 1:00pm in The Oil Drum: Europe
Topic: Supply/Production
Tags: logistic, oil production, oil reserves [list all tags]

This is a guest article by Dudley Stark, Reader in Mathematics and Probability in the School of Mathematical Sciences, Queen Mary, University of London. It is a summary of his paper The limit of the statistic R/P in models of oil discovery and production, which has been accepted to appear in the International Journal of Pure and Applied Mathematical Sciences.

> *"2005 was a third consecutive year of rising energy prices. Tight capacity, extreme weather, continued conflict in the Middle East, civil strife elsewhere and growing interest in energy among financial investors led to rising prices", said Lord Browne, CEO of BP plc. "Although energy prices have increased, there has been no physical shortage of either oil or gas." According to the BP Statistical Review of World Energy 2006, oil holds a reserves-to-production ratio of 40 years, gas of some 65 years and coal of 162 years. With the advancement of technology, more energy resources will also be discovered in the future."*
>
> Quote taken from the BP China Website. (Note: Lord Browne is no longer CEO of BP.)

The R/P statistic equals current reserves divided by the current rate of production. Reserves-to-production ratios R/P are sometimes used by companies to reassure us of the future. It would seem that R/P is supposed to represent a estimate of the amount of time for which production won't pose a problem.

My intention is to show that R/P can converge to a positive real number, or even diverge to infinity, as time tends to infinity. This happens when production decreases quickly enough with respect to the depletion of reserves. In particular, we will see that R/P converges to a positive real number when discovery and production curves are determined by the often used Logistic distributions. In a significant sense, this means that R/P cannot be a good measure of oil reserves; the time remaining for oil production cannot converge to a positive number and be a meaningful measure of time remaining for production.

Here are the R/P ratios for the top 10 oil producing nations, from April 2000, according to which the USA had R/P equal to 7, the UK had R/P equal to 5, and both countries should be out of oil by now. From International Energy Outlook 2007:

> The most common measure of the adequacy of proved reserves relative to annual production is the reserve-to production (r/p) ratio, which describes the number of years of remaining production from current proved reserves at current production rates. For the past 25 years, the U.S. r/p ratio has been between 9 and 12 years, and the top 40 countries in conventional crude oil production rarely have reported r/p ratios below 8 years. The major oil-producing countries of OPEC have maintained r/p ratios of 20 to

100 years.

In order to have a mathematical description of R/P, we must have models of both discovery and production.

Let $G_X(t)$ be the total amount of oil discovered by time t. Let $C=G_X(\text{infinity})$ be the total amount of oil ever discovered. Then $f_X(t)=G'_X(t)/C$ is the probability density function of some random variable X (hence the subscript X): it is nonnegative and its integral over the entire real line equals 1.

Similarly, if $G_Y(t)$ is the total amount of oil produced by time t, then $f_Y(t)=G'_Y(t)/C$ is the probability density function of some random variable Y (assuming that all oil discovered is produced).

The R/P statistic is given by $R/P=(G_X(t)-G_Y(t))/G'_Y(t)$. Note that (limit as t goes to infinity) $G_X(t)-G_Y(t)=C-C=0$. If we assume that (limit as t goes to infinity)$G'_Y(t)=0$, then we can apply L'Hospital's Rule from calculus and conclude that $R/P=[(G_X'(t)/C-G_Y'(t))/C]/G''_Y(t)/C$ $=(f_X(t)-f_Y(t))/f'_Y(t)$. The reason we have done this is because we will define $f_X(t)$ and $f_Y(t)$ explicitly; the functions $G_X(t)$ and $G_Y(t)$ are basically given by integrals and are harder to work with.

Given the discovery curve $f_X(t)$, a natural way of defining the production curve $f_Y(t)$ is to say Y=aX+b where a>1 or a=1 and b>0. The parameter b shifts the discovery curve and the parameter a rescales it. Here's a post on The Oil Drum about a paper of Pickering which establishes empirically that Y=aX+b for various countries. The requirement that a>1 or a=1 means that it takes at least as long to produce oil than to discover it. The production curve is then given by $f_Y(t)=f_X((t-b)/a)/a$. (In order to ensure that $G_Y(t) < G_X(t)$, so that oil is produced after it's discovered, I used a slightly different definition than the one here.) Thus, we have a framework where all we have to do is specify the distribution of X and the parameters a and b.

Figure 1 shows plots for discovery (red) and production (green) curves when $f_X(t)=(2*pi)^{-1/2}e^{-x^2/2}$ (the density of the standard normal distribution), a=2 and b=3 (so Y is Normal with mean 3 and variance 4).

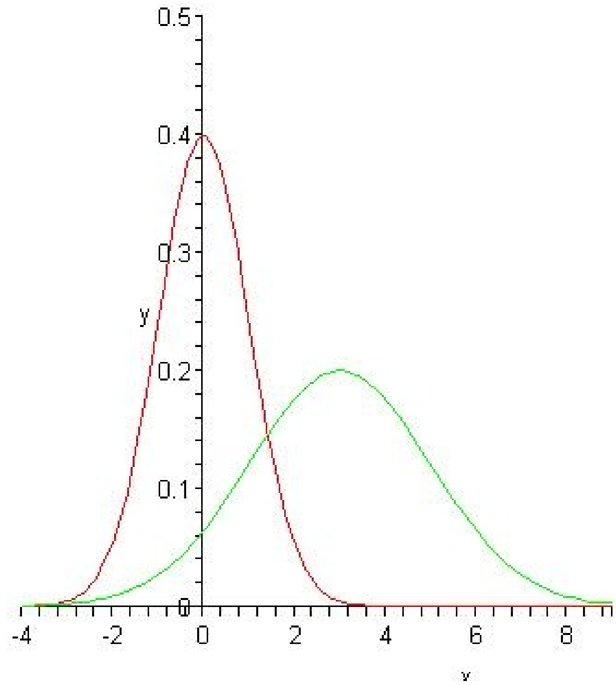

Figure 1

I find (limit to tends to infinity)R/P for the three distributions considered by Deffeyes in his book Hubbert's Peak: Normal (red), Logistic (green) and Lorentzian (cyan). (Lorentzian is also called Cauchy). I won't ask you to read the densities of these distributions in html. Three Normal, Logistic and Lorentzian distributions are plotted in figure 2. I have chosen the parameters mu for all distributions equal to 0 and the other parameters sigma, s and r all equal 1. See my paper for definitions of parameters. Deffeyes has a different and probably better way of choosing values of parameters for comparison in his book.

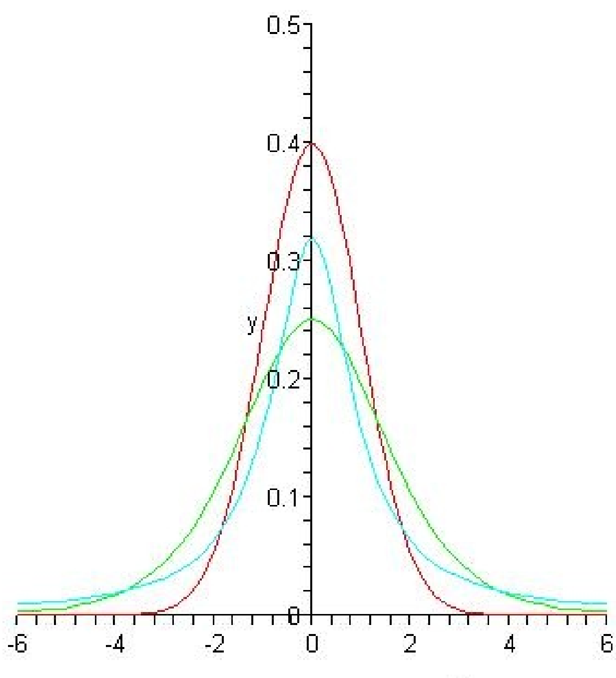

Figure 2

The important observation for our analysis is that the normal density decreases like e^{-At^2} the logistic distribution decreases like e^{-Bt}, and the Lorentzian distribution decreases like C/t^2 as t tends to infinity for some positive constants A, B and C. These different behaviours, the "tail behaviours" of the distributions, determine the behaviour of R/P as t tends to infinity.

The main result of the paper is

limit of R/P as t tends to infinity =

> 0 if X is Normal
>
> $s(1-e^{-b/s})$ if X is Logistic and a=1
>
> as if X is Logistic and a>1
>
> b if X is Lorentzian and a=1
>
> infinity if X is Lorentzian and a>1

(The parameter s is use in defining the Logistic distribution.)

Thus, if the distributions are Normal, then R/P does converge to 0. If the distributions are Logistic, then R/P converges to a positive number. If the distributions are Lorentzian, then R/P converges to a positive number or even diverges to infinity. The result for the Logistic distribution when a=1 was obtained previously by Broto in his poster for an ASPO conference .

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